Even numbers

2, 4, 6, 8, 10, 12, …………
2 divides exactly into every even number.

Odd numbers

1, 3, 5, 7, 11, …………
2 doesn't divide exactly into odd numbers.

Square numbers

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$
$6^2 = 6 \times 6 = 36$
$7^2 = 7 \times 7 = 49$

The first 7 square numbers are: 1, 4, 9, 16, 25, 36, 49

Multiples

Multiples of a number are all of the numbers that appear in its times table.
The multiples of 4 are 4, 8, 12, 16…
The 5th multiple of 6 is 30

Factors

A factor is a number that divides exactly into another number.
The factors of 12 are:
1, 2, 3 ,4, 6, 12
The factors of 13 are 1 and 13

The Highest Common Factor is the highest factor that two numbers have in common.
e.g. factors of 12 are 1, 2, 3, 4, 6, 12 and factors of 20 are 1, 2, 4, 5, 10, 20
1, 2, 4 are common factors. **4 is the highest common factor**

The Lowest Common Multiple is the lowest number common to the times tables.
e.g. multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54….
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64…
24 and 48 are common multiples. **24 is the lowest common multiple**

Triangular numbers

1	= 1
1 + 2	= 3
1 + 2 + 3	= 6
1 + 2 + 3 + 4	= 10
1 + 2 + 3 + 4 + 5	= 15
1 + 2 + 3 + 4 + 5 + 6	= 21
1 + 2 + 3 + 4 + 5 + 6 + 7	= 28

The first seven triangular numbers are:
1, 3, 6, 10, 15, 21, 28

Prime numbers

A prime number has exactly **two** factors namely 1 and itself.

The factors of 17 are 1 and 17, therefore 17 is a prime number.

The prime numbers between 1 and 30 are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Note: 1 is not a prime number!

Place value

Thousands (1000)	Hundreds (100)	Tens (10)	Units (1)	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$

10 units	=	1 ten
10 tens	=	1 hundred
10 hundreds	=	1 thousand

10 thousandths	=	1 hundredth
10 hundredths	=	1 tenth
10 tenths	=	1 unit

The placement of the digits within the number gives us the value of that digit.

The digit 4 has the value of 4 thousand (4000)

The digit 5 has the value of 5 tenths ($^5/_{10}$)

4 2 8 4 . 5 6 7

The digit 8 has the value 8 tens (80)

The digit 7 has the value 7 thousandths ($^7/_{1000}$)

Inverse operations

Inverse operations allow you to undo a sum.

Operator	Inverse Operator
+	−
−	+
÷	×
×	÷

We use inverse operations when we work with function machines.

Input ? → ÷ 3 → - 7 = 3 Output

If the output is 3, the input ? must be 30.

30 = × 3 ← + 7 ← 3

Fractions

The **numerator** is the number on the top of the fraction

$$\frac{3}{4}$$

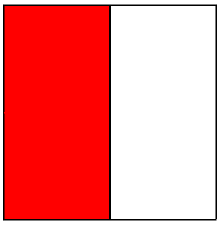
The **denominator** is the number on the bottom

If we have a number and a fraction mixed we call it a **mixed fraction**. $3\frac{7}{8}$

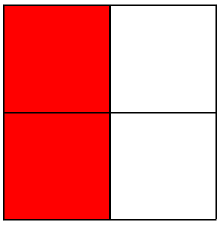
When the numerator is larger than the denominator we call this an **improper** fraction. $\frac{9}{7}$

Equivalent fractions

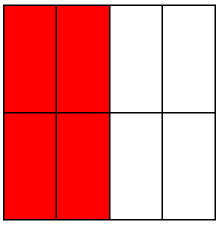
All the fractions below represent the same proportion. Therefore they are called equivalent fractions.



$^1/_2$



$^2/_4$



$^4/_8$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \ldots$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \ldots$$

etc.

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \ldots$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \ldots$$

Decimals

A decimal is any number that contains a decimal point.
The following are examples of decimals.

0·549 1·25 256·4 3·406

Percentages

The symbol % means $\frac{1}{100}$

7%	means	$^{7}/_{100}$	
63%	means	$^{63}/_{100}$	
100%	means	$^{100}/_{100}$	or 1 whole.
120%	means	$^{120}/_{100}$	It is possible to have a percentage that is greater than 1 whole.

Changing decimals and fractions into percentages

To change a decimal or fraction to a percentage you have to multiply with 100%.

$$0{\cdot}75 \times 100\% = 75\%$$

$$\frac{13}{{}_{1}\cancel{20}} \times {}^{5}\cancel{100}\% = 65\%$$

To change a fraction into a decimal you have to divide the numerator with the denominator.

$$\frac{3}{8} = 3 \div 8 = 0{\cdot}375$$

It is also possible to change a fraction into a percentage like this:

$$\frac{2}{3} = 2 \div 3 = 0{\cdot}6666\ldots = 0{\cdot}67 \text{ (to 2 decimal places)}$$

$$\text{then } 0{\cdot}67 \times 100\% = 67\%$$

Therefore $\frac{2}{3}$ **= 67% (to the nearest one part of a hundred)**

Useful fractions, decimals and percentages

Fraction	Decimal	Percentage
1	1·0	100%
$^{1}/_{2}$	0·5	50%
$^{1}/_{3}$	0·33.....	33%
$^{1}/_{4}$	0·25	25%
$^{3}/_{4}$	0·75	75%
$^{1}/_{10}$	0·1	10%
$^{2}/_{10}$ (= $^{1}/_{5}$)	0·2	20%
$^{3}/_{10}$	0·3	30%

Ratio

Ratio is used to make a comparison between two things.

Example

☺ ☺ ☺ ☹ ☺ ☺ ☺ ☹ ☺ ☺ ☺ ☹

In this pattern we can see that there are 3 happy faces **to** every sad face.
We use the symbol **:** to represent **to** in the above statement, therefore we write the ratio like this:

Happy	:	Sad		Sad	:	Happy
3	:	1		1	:	3

Ratio is used in a number of situations:

- In a cooking recipe
- In building when mixing concrete
- It is used in the scale of maps
  e.g. if a scale of **1 : 100 000** is used,
  it means that **1 cm** on the map represents
  100 000 cm in reality which is **1 km**.

Directed numbers

The negative sign (-) tells us the number is below zero e.g. **-4**. The number line is useful when working with negative numbers. Below is a part of the number line.

Negative direction ← → Positive direction

-9	-8	-7	-6	-5	**-4**	-3	-2	-1	0	1	2	3	4	5	6

The numbers on the right are greater than the numbers on the left e.g. 5 is greater than 2 and 2 is greater than -3. **Note** that -3 is greater than -8.

Adding and subtracting directed numbers

The **Number line game** can be used to add and subtract negative numbers:

Rules:

Start at zero facing the positive direction.
Ignore any **+** signs.
The **–** sign means **"make half a turn"** .
When you see a number, step the value of the number in the direction you are facing.
After stepping, face the positive direction before continuing with the sum.
Your position at the end will be the answer.

Example: **- 3 – 4 + 6**

Sum	-8	-7	-6	-5	-4	-3	-2	-1	0	1	Method
									→		Start at zero.
-									←		Make half a turn.
3						←					Step 3.
						→					Face the positive.
-						←					Make half a turn.
4		←									Step 4.
		→									Face the positive.
+		→									Ignore the +.
6								→			Step 6.
								☺			**The answer is -1.**

Example: 2 + - 8 - - 9

- Start at zero facing the positive direction.
- Step 2 and face the positive direction.
- Ignore the + , make half a turn, step 8 and face the positive direction.
- Make half a turn, make half a turn, step 9 and note your position.
  The answer is **3** :

Multiplying and dividing directed numbers

We multiply and divide directed numbers in the usual way whilst remembering these very important rules:

Two signs the same, a positive answer.
Two different signs, a negative answer.

×	+	–
+	+	–
–	–	+

÷	+	–
+	+	–
–	–	+

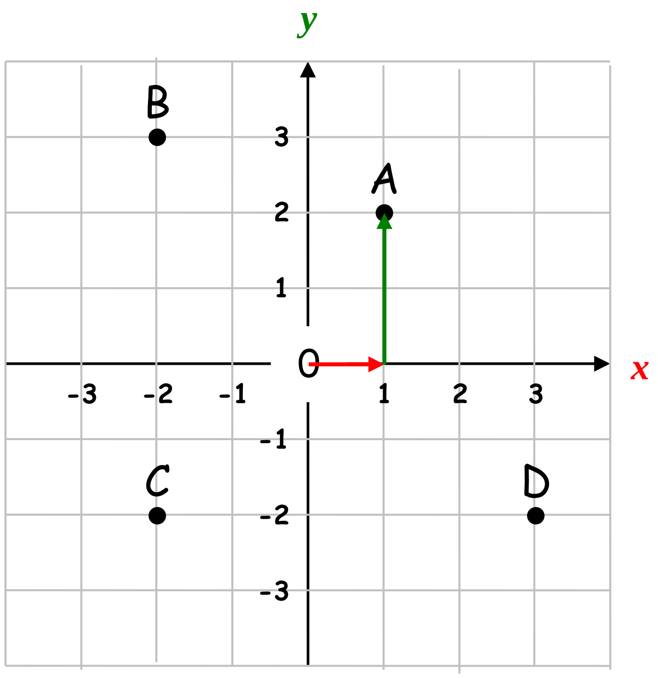
Remember, if there is no sign before the number, it is positive.

Examples:

5	**x**	**-7**	**=**	**-35**	**(different signs give a negative answer)**
-4	**x**	**-8**	**=**	**32**	**(two signs the same give a positive answer)**
48	**÷**	**-6**	**=**	**-8**	**(different signs give a negative answer)**
-120	**÷**	**-10**	**=**	**12**	**(two signs the same give a positive answer)**

Coordinates

We use coordinates to describe location.



The coordinates of the points are:

A(1,2) **B(-2,3)** **C(-2,-2)** **D(3,-2)**

There is a special name for the point **(0,0)** which is **the origin**.

The first number (**x-coordinate**) represents the distance across from the origin.
The second number (**y-coordinate**) represents the distance going up or down.

Example : The point (1,2) is **one across** and **two up** from the origin.

Inequalities

We us the = sign to show that two sums are **equal**. If one sum is greater than or less than the other we use inequalities:

$<$ less than

$>$ more than

$\leq$ less than or equal to

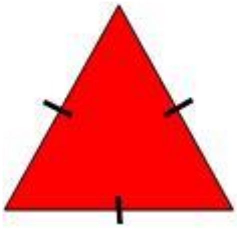
$\geq$ more than or equal to

Examples :

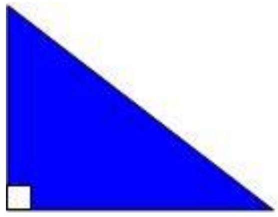
$5 < 8$ $43 > 6$ $x \leq 8$ $y \geq 17$

Names of two dimensional shapes

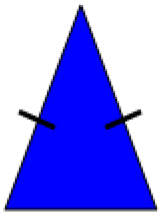
A polygon is a closed shape made up of straight lines.
A **regular polygon** has equal sides and equal angles.



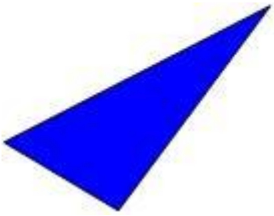
Equilateral triangle



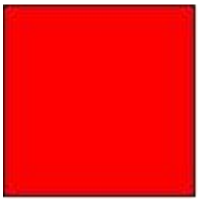
Right angled triangle



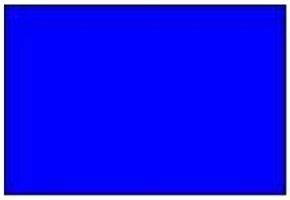
Isosceles triangle



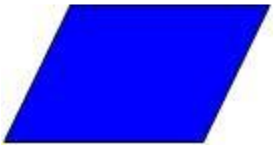
Scalene triangle



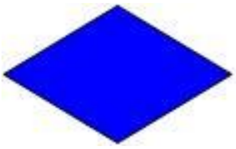
Square



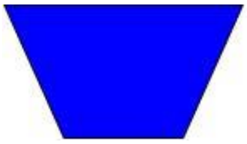
Rectangle



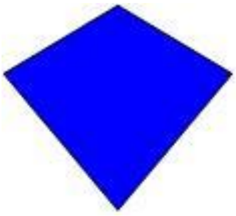
Parallelogram
Opposite sides parallel and equal.



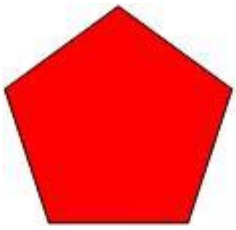
Rhombus
Opposite sides parallel, all sides equal.



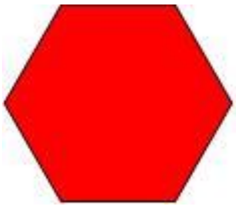
Trapezium
One pair of opposite sides parallel.



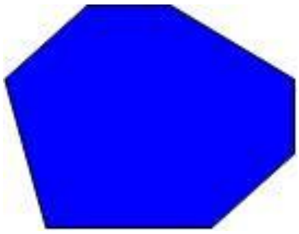
Kite



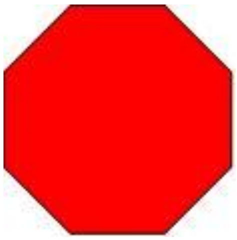
Pentagon



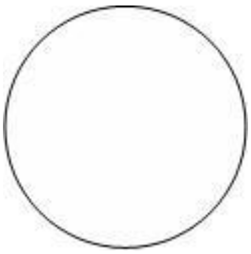
Hexsagon



Heptagon



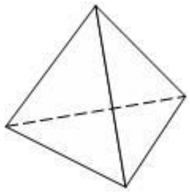
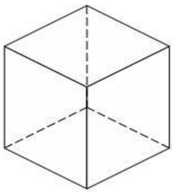
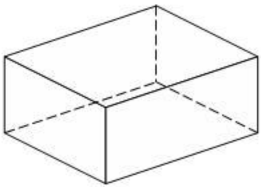
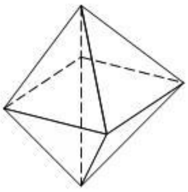
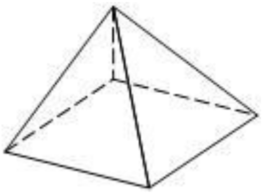
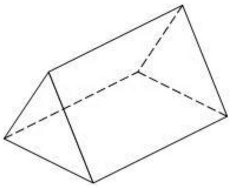
Octagon



Circle

3D shapes

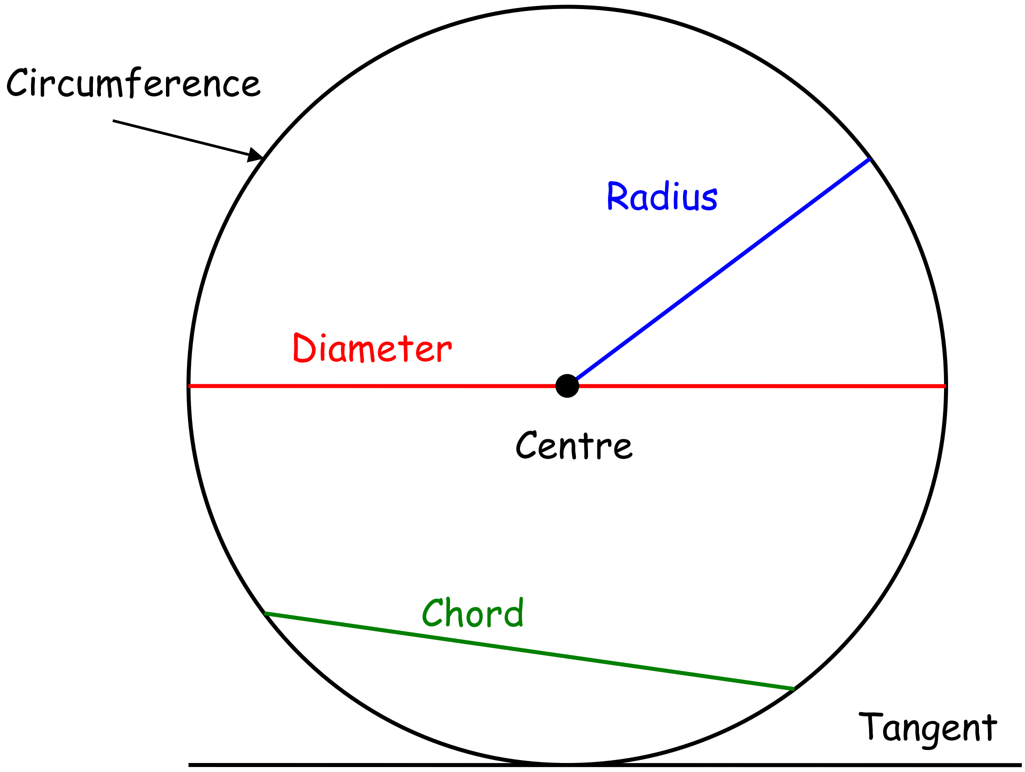
3D means three dimensions – 3D shapes have length, width and height.

Shape	Name	Faces	Edges	Vertices (corners)
	Tetrahedron	4	6	4
	Cube	6	12	8
	Cuboid	6	12	8
	Octahedron	8	12	6
	Square based pyramid	5	8	5
	Triangular prism	5	9	6

Euler's formula:

Number of faces - Number of edges + Number of vertices = 2

The circle



Circumference of a circle

The circumference of a circle is the distance around the circle.

Circumference = π x diameter

Circumference = πd

Since the the diameter is **twice** the length of the radius, we can also write

Circumference = π x 2 x radius

Circumference = $2\pi r$

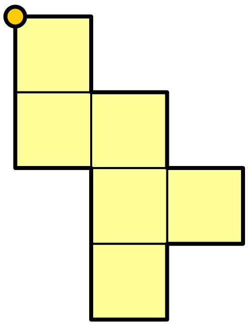
π (pi)

π is a Greek letter which represents 3·14159265358979323 84 . . . . . (a decimal that carries on for ever without repetition)

We round π to 3·14 in order to make calculations or we use the π button on the calculator.

Perimeter

Perimeter is the distance around the outside of a shape. We measure the perimeter in millimetres (mm), centimetres (cm), metres (m), etc.



This shape has been drawn on a 1cm grid. Starting on the orange circle and moving in a clockwise direction, the distance travelled is . . .

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 1 + 2 = 14cm

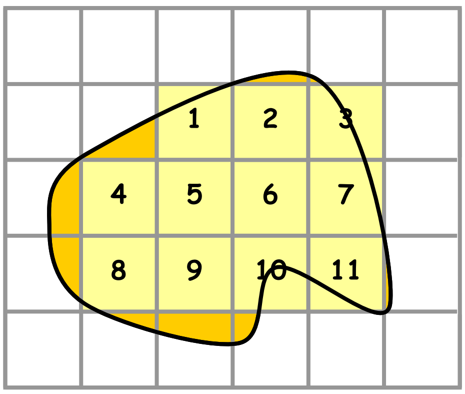
Perimeter = 14cm

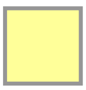
Area of 2D Shapes

The area of a shape is how much surface it covers. We measure area in square units e.g. centimetres squared (cm^2) or metres squared (m^2).

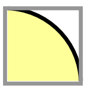
Areas of irregular shapes

Given an irregular shape, we estimate its area through drawing a grid and counting the squares that cover the shape.

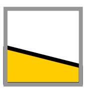




Whole square – count as one.



Half a square or more – count as one.



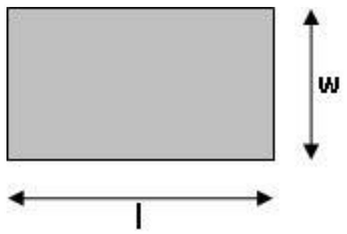
Less than half a square - ignore.

Area = $11cm^2$.

Remember that this is an estimate and not the exact area.

Area formulae

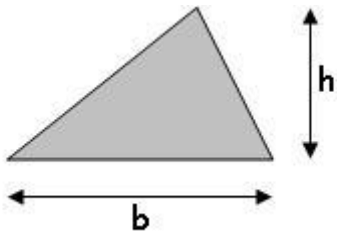
Rectangle



Multiply the length with the width.

Area = $l \times w$

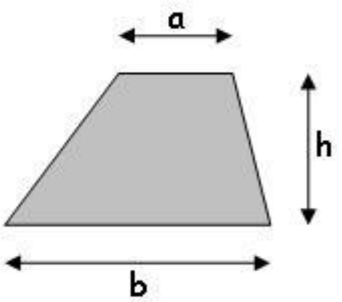
Triangle



Multiply the base with the height and divide by two.

Area = $\frac{b \times h}{2}$

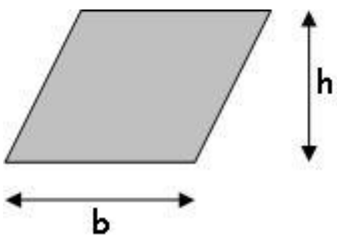
Trapezium



Add the parallel sides, multiply with the height and divide by two.

Area = $\frac{(a + b)\,h}{2}$

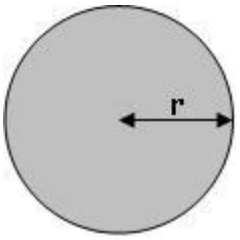
Parallelogram



Multiply the base with the height.

Area = $b \times h$

Circle



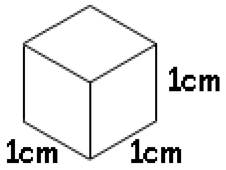
Multiply the radius with itself, then multiply with π.

Area = $r \times r \times \pi = \pi r^2$

Volume

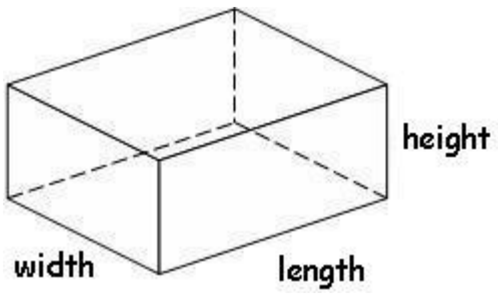
Volume is the amount of space that an object contains or takes up. The object can be a solid, liquid or gas.

Volume is measured in cubic units e.g. cubic centimetres (cm^3) and cubic metres (m^3).



Cuboid

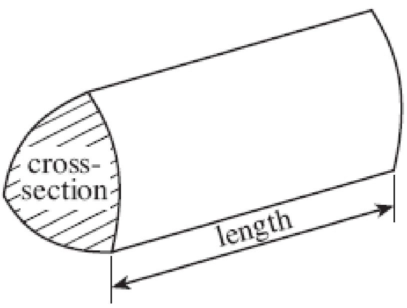
Note that a cuboid has six rectangular faces.



Volume of a cuboid = length x width x height

Prism

A prism is a 3-dimensional object that has the same shape throughout its length i.e. it has a uniform cross-section.



Volume of a prism = area of cross-section x length

Metric units of length

Millimetre	mm	10 mm = 1 cm 1 000 mm = 1 m
Centimetre	cm	100 cm = 1 m 100 000 cm = 1 km
Metre	m	1 000 m = 1 km
Kilometre	km	

Imperial units of length

Inch	in or "	12 in = 1 ft
Foot	ft or '	3 ft = 1 yd
Yard	yd	1 760 yd = 1 mile
Mile		

Metric units of mass

Milligram	mg	1 000 mg = 1 g 1 000 000 mg = 1 kg
Gram	g	1 000 g = 1 kg
Kilogram	kg	1 000 kg = 1 t
Metric tonne	t	

Imperial units of mass

Ounce	oz	16 oz = 1 lb
Pound	lb	14 lb = 1 st
Stone	st	160 st = 1 t

Metric units of volume

Millilitre	ml	1 000 ml = 1 l
Litre	l	

Imperial units of volume

Pint	pt	8 pt = 1 gal
Gallon	gal	

Converting between imperial and metric units

Length

1 inch	≈	2.5 cm
1 foot	≈	30 cm
1 mile	≈	1.6 km
5 miles	≈	8 km

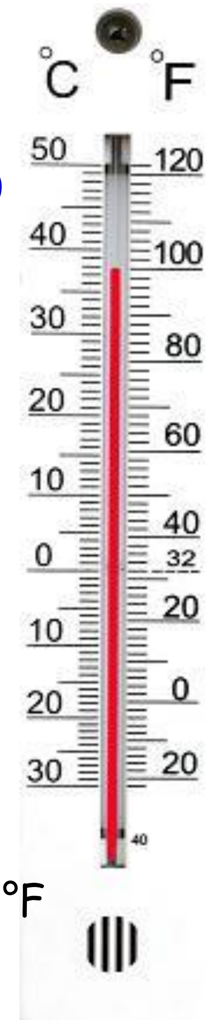
Weight/Mass

1 pound	~	454 g
2.2 pounds	~	1 kg
1 ton	~	1 metric tonne

Volume

1 gallon	≈	4.5 litre
1 pint	≈	0.6 litre(568 ml)
$1\frac{3}{4}$ pints	≈	1 litre

Temperature



Converting from Celsius (°C) to Fahrenheit (°F)

Use the following formula

$$F = 1.8 \times C + 32$$

Converting from Fahrenheit (°F) to Celsius (°C)

Use the following formula

$$C = (F - 32) \div 1.8$$

Look at the thermometer:

The freezing point of water is 0°C or 32°F

Time

1000	years	=	1 millennium
100	years	=	1 century
10	years	=	1 decade
60	seconds	=	1 minute
60	minutes	=	1 hour
24	hours	=	1 day
7	days	=	1 week
12	months	=	1 year
52	weeks	≈	1 year
365	days	≈	1 year
366	days	≈	1 leap year

The Yearly Cycle

Season	Month	Days
	January	31
	February	28
	March	31
	April	30
	May	31
	June	30
	July	31
	August	31
	September	30
	October	31
	November	30
	December	31

The 24 hour and 12 hour clock

	24 hour		12 hour	
Midnight	00:00		12.00 a.m.	Midnight
The 24 hour clock always uses 4 digits to show the time. **The 24 hour system does not use a.m. nor p.m.**	01:00		1:00 a.m.	The 12 hour clock shows the time with a.m. before mid-day and p.m. after mid-day.
	02:00		2:00 a.m.	
	03:00		3:00 a.m.	
	04:00		4.00 a.m.	
	05:00		5:00 a.m.	
	06:00		6:00 a.m.	
	07:00		7:00 a.m.	
	08:00		8:00 a.m.	
	09:00		9:00 a.m.	
	10:00		10:00 a.m.	
	11:00		11:00 a.m.	
Mid-day	12:00		12:00 p.m.	Mid-day
	13:00		1:00 p.m.	
	14:00		2:00 p.m.	
	15:00		3:00 p.m.	
	16:00		4:00 p.m.	
	17:00		5:00 p.m.	
	18:00		6:00 p.m.	
	19:00		7:00 p.m.	
	20:00		8:00 p.m.	
	21:00		9.00 p.m.	
	22:00		10.00 p.m.	
	23:00		11:00 p.m.	

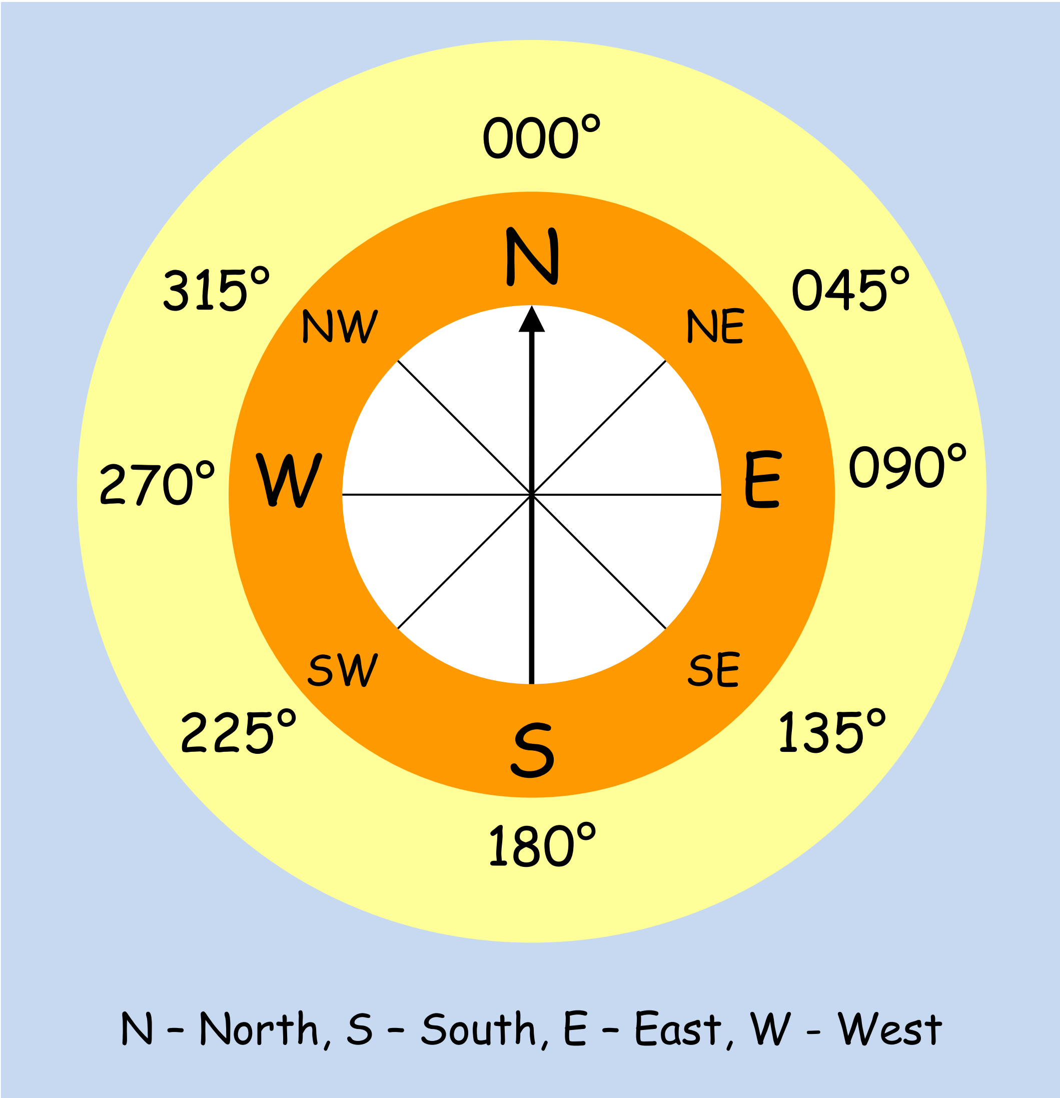
Time vocabulary

02:10	Ten past two in the morning	2:10 a.m.
07:15	Quarter past seven in the morning	7:15 a.m.
15:20	Twenty past three in the afternoon	3:20 p.m.
21:30	Half past nine in the evening	9:30 p.m.
14:40	Twenty to three in the afternoon	2:40 p.m.
21:45	Quarter to ten at night	9:45 p.m.

Bearings

A bearing describes direction. A compass is used to find and follow a bearing.

The diagram below shows the main compass points and their bearings.



The bearing is an angle measured clockwise from the North.

Bearings are always written using three figures e.g. if the angle from the North is 5°, we write 005°.

Data

We collect data in order to highlight information to be interpreted.

There are two types of data:

Discrete data Things that are not measured:	**Continuous data** Things that are measured:
• Colours • Days of the week • Favourite drink • Number of boys in a family • Shoe size	• Pupil height • Volume of a bottle • Mass of a chocolate bar • Time to complete a test • Area of a television screen

Discrete data

Collecting and recording

We can record data in a list

e.g. here are the numbers of pets owned by pupils in form 9C:

1 , 2 , 1 , 1 , 2 , 3 , 2 , 1 , 2 , 1 , 1 , 2 , 4 , 2 , 1 , 5 , 2 , 3 , 1 , 1 , 4 , 10 , 3 , 2 , 5 , 1

A frequency table is more structured and helps with processing the information.

Number of pets	Tally	Frequency
1	𝍸 𝍸	10
2	𝍸 |||	8
3					3
4				2
5				2
6		0
7		0
8		0
9		0
10			1

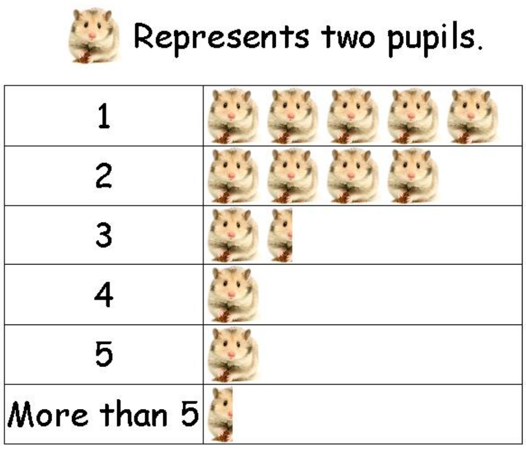
Displaying

In order to communicate information, we use statistical diagrams. Here are some examples:

Pictogram

A pictogram uses symbols to represent frequency. We include a key to show the value of each symbol.

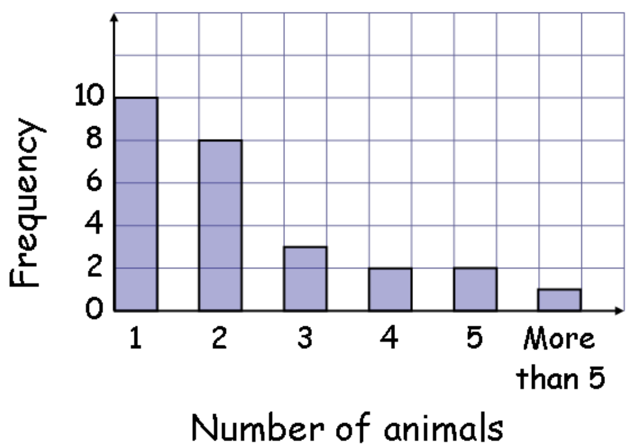
The diagram below shows the number of pets owned by pupils in 9C.



Bar chart

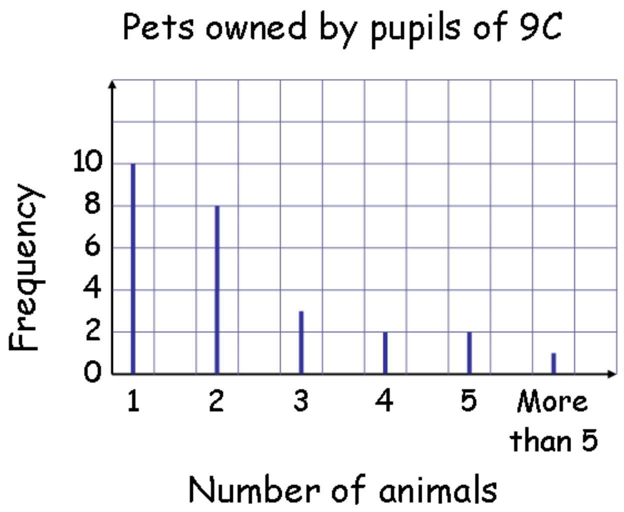
The height of each bar represents the frequency. All bars must be the same width and have a constant space between them. Notice that the scale of the frequency is constant and starts from 0 every time. Remember to label the axes and give the chart a sensible title.

Pets owned by pupils of 9C



Vertical line graph

A vertical line graph is very similar to a bar chart except that each category has a line instead of a bar. Notice that the category labels are directly below each line.



Pie chart

The complete circle represents the total frequency. The angles for each sector are calculated as follows:

Here is the data for the types of pets owned by 9C

Type of pet	Frequency	Angle of the sector
Cats	13	13 × 10° = 130°
Dogs	11	11 × 10° = 110°
Birds	5	5 × 10° = 50°
Fish	7	7 × 10° = 70°
Total	36	360°

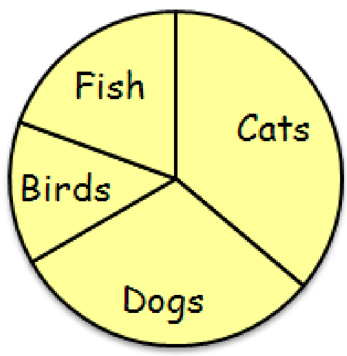
Divide 360° by the total of the frequency:

360° ÷ 36 = 10°

Therefore 10° represents one animal

Remember to check that the angles of the sectors add up to 360°.

Types of pet owned by 9C



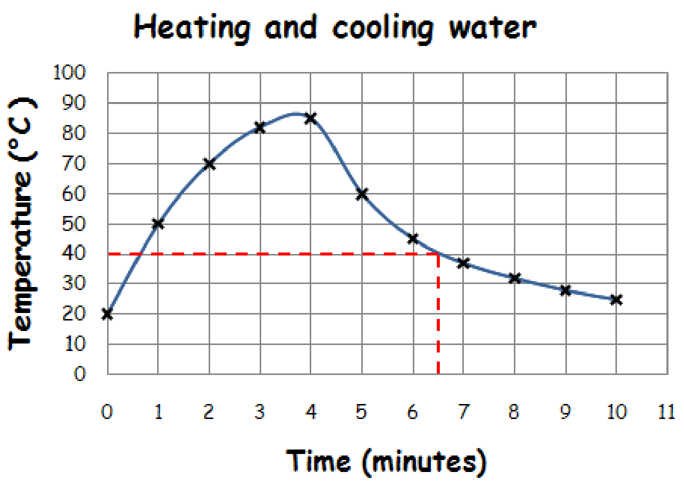
Continuous data

Displaying

With graphs representing continuous data, we can draw lines to show the relationship between two variables. Here are some examples:

Line graph

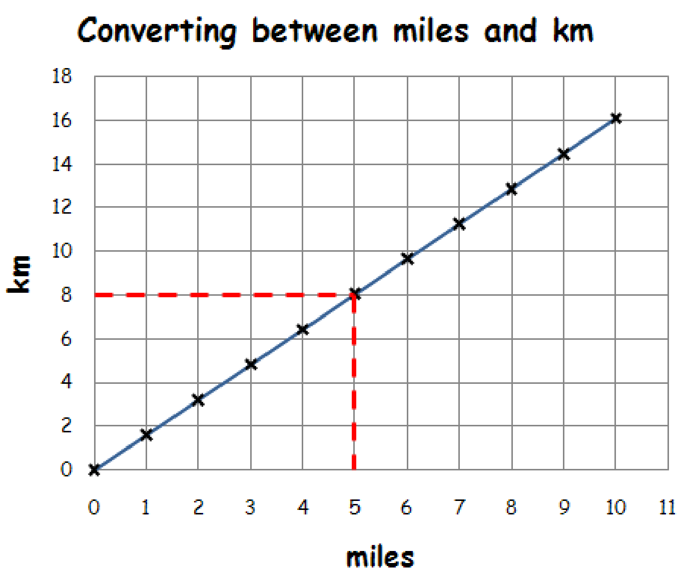
The temperature of water was measured every minute as it was heated and left to cool. A cross shows the temperature of the water at a specific time. Through connecting the crosses with a curve we see the relationship between temperature and time.



The line enables us to estimate the temperature of the water at times other than those plotted e.g. at $6\frac{1}{2}$ minutes the temperature was approximately 40 °C.

Conversion graph

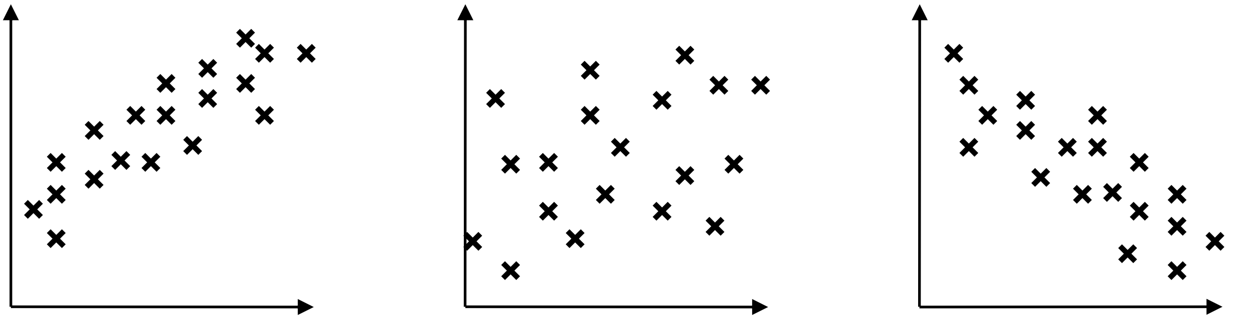
We use a conversion graph for two variables which have a linear relationship. We draw it in the same way as the above graph but the points are connected with a straight line.



From the graph, we see that 8 km is approximately 5 miles.

Scatter diagram

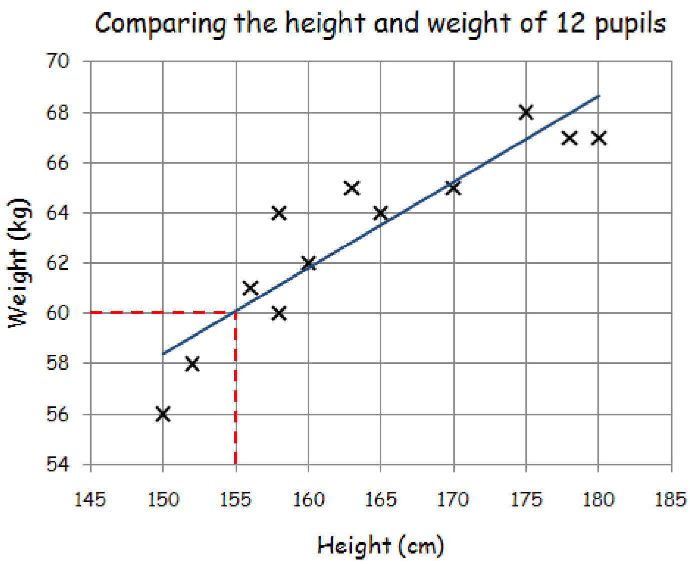
We plot points on the scatter diagram in the same way as for the line graph. We do not join the points but look for a correlation between the two sets of data.



Positive correlation No correlation Negative correlation

If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

The following scatter graph shows a positive correlation between the weights and heights of 12 pupils.



The line of best fit estimates the relationship between the two variables.
Notice that the line follows the trend of the points.
There are approximately the same number of points above and below the line.

We estimate that a pupil 155 cm tall has a weight of 60 kg.

Important things to remember when drawing graphs

- Title and label axes
- Sensible scales
- Careful and neat drawing with a pencil

Average

The average is a measure of the middle of a set of data. We use the following types of average:

Mean - We add the values in a set of data, and then divide by the number of values in the set.

Median - Place the data in order starting with the smallest then find the number in the middle. This is the median. If you have two middle numbers then find the number that's halfway between the two.

Mode - This is the value that appears most often.

Spread

The spread is a measure of how close together are the items of data. We use the following to measure spread:

Range - The range of a set of data is the difference between the highest and the lowest value.

Example

Find the mean, median, mode, and range of the following numbers:

4 , 3 , 2 , 0 , 1 , 3 , 1 , 1 , 4 , 5

Mean $\dfrac{4+3+2+0+1+3+1+1+4+5}{10} = 2{\cdot}4$

Median 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 $\dfrac{2+3}{2} = 2{\cdot}5$

Mode 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 = 1

Range 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 5 - 0 = 5